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# Fermion mass effects in $e^+e^- \to 4f$ and $e^+e^- \to 4f\gamma$ with cuts*

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**Abstract.** The fermion mass effects in $e^+e^- \to 4f$ and in the corresponding bremsstrahlung reactions in the presence of realistic cuts are studied. It is shown that, for some four–fermion final states, the mass effects become sizable to the extent that they may affect the accuracy of theoretical predictions which is required to be better than 1%.

## 1 Introduction

Before the recent shutdown of LEP the measurement of the $W$-boson-pair production cross-section at LEP 2 had reached an accuracy of 1% which required the inclusion of higher order corrections in the comparison with the Standard Model (SM) theoretical prediction [1,2]. At future $e^+e^-$–colliders, and in particular at a high luminosity machine like the high energy superconducting linear accelerator TESLA, proposed in [3], it will be necessary to provide theoretical predictions at a precision of about 0.1%. Such high precision of the theoretical predictions can only be achieved by including the complete set of electroweak radiative corrections at the one-loop level as well as the leading electroweak logarithms at higher-loops of the Standard Model.

A calculation of the complete set of the SM one-loop virtual corrections to $e^+e^- \to 4f$, which are reactions of actual interest, is a very tedious task, and despite the fact that some progress in calculating the corrections to $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ was reported in [4], at present there is no final result available for any of the possible four–fermion final states. Fortunately, a theoretical precision which is satisfactory for most applications in the analysis of the LEP2 data, could be achieved within the double-pole approximation (DPA) [5]. Recently, an interesting complete analysis of the virtual and real photonic corrections in the DPA has been reported in [6]. In [6], the DPA has been applied actually only to the non-leading virtual $\mathcal{O}(\alpha)$ corrections while real photonic corrections have been obtained with the full matrix elements of $e^+e^- \to 4f\gamma$ in the massless fermion limit. A great advantage of the double-pole approximation is that its basic ingredients such as the production of the on-shell W-pairs [7] and W-boson decay [8] have already been calculated at one-loop in the SM. Also the so called non-factorizable virtual corrections are known to have a simple structure and can be found in the literature [9].

Real photon corrections and in particular the hard bremsstrahlung are inherent ingredients of the $\mathcal{O}(\alpha)$ radiative corrections to the four–fermion processes and precise treatment of them is crucial for the ultimate accuracy of theoretical predictions which, for a proper analysis of the future $e^+e^-$ linear collider data, should possibly match the level of 0.1%. At the moment, there exist several packages which allow one to calculate cross sections of $e^+e^- \to 4f\gamma$ for any possible final state. They have been compared in [2]. Three of the codes based on full matrix elements: `WRAP, RacoonWW and PHEGAS/HELAC` have been subject to tuned comparisons in the approximation of massless fermions in the presence of cuts and they show a very good agreement for the final states considered in [2]. A complete list of results for total cross sections of all representative processes $e^+e^- \to 4f\gamma$ can be found in [10].

The fermion mass effects for $e^+e^- \to 4f$ and $e^+e^- \to 4f\gamma$ for different $CC10$ final states have been studied in [11] for the total cross sections without cuts, except for the photon energy cut. It has been shown that the cross sections of $e^+e^- \to 4f\gamma$ differ by up to a few per cent for final states including particles with different masses. The natural explanation for it is that the collinear divergences are regularized by different fermion masses in each case and therefore a small change in the fermion mass results in sizable effects in the total cross sections. One would not expect any numerically sizable effects of the fermion masses in the presence of angular and invariant mass cuts, as the angles between particles are relatively big then. However, as it will be demonstrated in the following, the massless fermion limit which is usually used in the non-

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collinear phase space region, may be a source of inaccuracy which may substantially affect the desired accuracy level of 0.1%.

## 2 The calculation

In the present section we sketch the basics of the calculation. We refer to [12] and [11] for more details.

The matrix elements of the reactions considered in the present paper are calculated with the helicity amplitude method. Parts of the Feynman graphs which contain a single uncontracted Lorentz index are defined as generalized polarization vectors and used as building blocks of the amplitudes. Particular care is taken of the photon radiation off the external fermion lines. The corresponding fermion propagators are expanded in the light fermion mass analytically in order to avoid numerical cancellations. Fermion masses are kept nonzero both in the kinematics and in the matrix elements. Keeping the nonzero fermion masses allows for the proper treatment of the collinear photons. Therefore cross sections can be calculated independently of angular cuts and the background from undetected hard photons can be estimated. Moreover, a photon exchange in the $t$-channel can be handled better than in the massless fermion limit and the Higgs boson effects can be incorporated consistently.

The photon propagator is taken in the Feynman gauge and the propagators of the massive gauge bosons $W^\pm$ and $Z^0$, are defined in the unitary gauge. The constant widths $\Gamma_W$ and $\Gamma_Z$ are introduced through the complex mass parameters

$$M_V^2 = m_V^2 - i m_V \Gamma_V \tag{1}$$

in the propagators. However, the electroweak mixing parameter is kept real, although there is no obstacle to having it complex. This simple prescription preserves the electromagnetic gauge invariance with the nonzero fermion masses, even when the widths $\Gamma_W$ and $\Gamma_Z$ are treated as two independent parameters, which has been checked numerically, and for some final states, also analytically.

The constant width prescription violates the $SU(2)$ gauge-symmetry. However, the corresponding numerical effects caused by spoiling the gauge cancellations are, in the presence of cuts, practically irrelevant up to the relatively high c.m.s. energy of 10 TeV. This observation relies on the comparison with the results of [10]. Our results for $e^+e^- \to u\bar{d}e^-\bar{\nu}_e$, $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ and the corresponding bremsstrahlung reactions, which were calculated in the linear gauge, agree within statistical errors with those of [10] which were obtained in a nonlinear gauge in the so called complex-mass scheme that preserves the Ward identities.

The phase space integration is performed numerically. The 7 (10) dimensional phase space element of $e^+e^- \to 4f$ ($e^+e^- \to 4f\gamma$) is parametrized in several different ways, which are combined in a single multichannel Monte Carlo (MC) integration routine. In the soft photon limit, the photon phase space is integrated analytically.

The most relevant peaks of the matrix elements, e.g., the collinear peaking related to the initial and final state radiation, the $\sim 1/t$ pole caused by the $t$-channel photon-exchange, the $\sim 1/k$ peaking of the bremsstrahlung photon spectrum, the Breit–Wigner shape of the $W^\pm$ and $Z^0$ resonances, the $\sim 1/s$ behavior of a light fermion pair production, and the $\sim 1/t$ pole due to the the neutrino exchange are mapped away before applying the MC integration routine `VEGAS` [13].

## 3 Numerical results

In this section, we will present numerical results for several different four–fermion reactions $e^+e^- \to 4f$ and the corresponding bremsstrahlung reactions.

We define the set of physical parameters, as in [2], by the gauge boson masses and widths:

$$\begin{aligned} m_W &= 80.35 \text{ GeV}, \quad \Gamma_W = 2.08699 \text{ GeV}, \\ m_Z &= 91.1867 \text{ GeV}, \quad \Gamma_Z = 2.49471 \text{ GeV}, \end{aligned} \tag{2}$$

by the couplings which are defined in terms of the electroweak mixing parameter $\sin^2\theta_W = 1 - m_W^2/m_Z^2$ and the fine structure constant at two different scales, $\alpha_W$ and $\alpha$, the latter being used for parametrization of couplings of the external photon,

$$\alpha_W = \sqrt{2} G_\mu m_W^2 \sin^2\theta_W/\pi, \quad \alpha = 1/137.0359895, \tag{3}$$

with $G_\mu = 1.16637 \times 10^{-5}$ GeV$^{-2}$.

Only for the sake of comparison with [10], consider a second set of parameters, originally proposed in [14], which is given again by the gauge boson masses and widths

$$\begin{aligned} m_W &= 80.23 \text{ GeV}, \quad \Gamma_W = 2.0337 \text{ GeV}, \\ m_Z &= 91.1888 \text{ GeV}, \quad \Gamma_Z = 2.4974 \text{ GeV}, \end{aligned} \tag{4}$$

the single value of the fine structure constant plus in addition the electroweak mixing parameter

$$\alpha_W = 1/128.07, \qquad \sin^2\theta_W = \pi\alpha_W/(\sqrt{2} G_\mu m_W^2), \tag{5}$$

with $G_\mu = 1.16639 \times 10^{-5}$ GeV$^{-2}$. It should be stressed at this point that the relation $M_W = M_Z \cos\theta_W$ is not valid any longer with the parameter set of (4) and (5). We only use it in Table 2, where we compare our results with those presented in [10]. Needless to say, one should avoid utilizing this kind of paramatrization as it assumes 4 free parameters, while only 3 of them are independent in the SM.

The charged lepton masses are given by

$$\begin{aligned} &m_e = 0.51099906 \text{ MeV}, \quad m_\mu = 105.658389 \text{ MeV}, \\ &m_\tau = 1777.05 \text{ MeV} \end{aligned} \tag{6}$$

and for the quark masses we take

$$\begin{aligned} &m_u = 5 \text{ MeV}, \quad m_d = 10 \text{ MeV}, \quad m_s = 150 \text{ MeV}, \\ &m_c = 1.5 \text{ GeV}, \quad m_b = 5 \text{ GeV}. \end{aligned} \tag{7}$$

**Table 1.** Bremsstrahlung cross sections $\sigma_s$, $\sigma_h$ and $\sigma_s+\sigma_h$ in fb for two different photon energy cuts $\omega$ and two c.m.s. energies of selected four–fermion reactions. No other kinematical cuts are imposed, except for $t_0$ discussed in the text for the first of presented final states. The photon mass is $m_\gamma = 10^{-6}$ GeV

| Final state | $\omega$ | $\sqrt{s}=190$ GeV | | | $\sqrt{s}=500$ GeV | | |
|---|---|---|---|---|---|---|---|
| | (GeV) | $\sigma_s$ | $\sigma_h$ | $\sigma_s+\sigma_h$ | $\sigma_s$ | $\sigma_h$ | $\sigma_s+\sigma_h$ |
| $\nu_\tau\tau^+e^-\bar{\nu}_e\gamma$ | $10^{-3}$ | 60.60(4) | 420.9(8) | 481.5 | 24.01(4) | 683.6 (2.2) | 707.6 |
| | $10^{-1}$ | 258.1(2) | 223.8(4) | 481.9 | 305.3(3) | 403.4(1.2) | 708.7 |
| $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu\gamma$ | $10^{-3}$ | 63.26(3) | 314.6(4) | 377.9 | 13.74(1) | 155.8(3) | 169.5 |
| | $10^{-1}$ | 210.8(1) | 167.2(2) | 378.0 | 73.46(4) | 96.0(2) | 169.5 |
| $\nu_\mu\bar{\nu}_\mu\tau^-\tau^+\gamma$ | $10^{-3}$ | 2.806(1) | 13.97(2) | 16.78 | 0.8071(6) | 6.193(11) | 7.001 |
| | $10^{-1}$ | 9.171(4) | 7.609(11) | 16.78 | 3.158(2) | 3.841(7) | 6.999 |

We define a set of cuts, identical to those of [10]

$$\begin{aligned}
&\theta(l,\text{beam}) > 10^\circ, \quad \theta(l,l') > 5^\circ, \quad \theta(l,q) > 5^\circ,\\
&\theta(\gamma,\text{beam}) > 1^\circ, \quad \theta(\gamma,l) > 5^\circ, \quad \theta(\gamma,q) > 5^\circ,\\
&E_\gamma > 0.1\ \text{GeV}, \quad E_l > 1\ \text{GeV}, \quad E_q > 3\ \text{GeV},\\
&m(q,q') > 5\ \text{GeV}\,,
\end{aligned} \tag{8}$$

where $l$, $q$, $\gamma$, and "beam" denote charged leptons, quarks, photons, and the beam (electrons or positrons), respectively, and $\theta(i,j)$ the angles between the particles $i$ and $j$ in the c.m.s. Furthermore, $m(q,q')$ denotes the invariant mass of a quark pair $qq'$.

We perform a number of checks. The matrix elements have been checked against `MADGRAPH` [15] and the phase space integrals have been checked against their asymptotic limits obtained analytically. The electromagnetic gauge invariance of the matrix element of the bremsstrahlung process has been checked numerically and for some final states also analytically.

The cut independence of the total bremsstrahlung cross section $\sigma_\gamma = \sigma_s + \sigma_h$ has been tested. $\sigma_s$ denotes the soft photon contribution to the cross section, which includes photons of energy $E_\gamma \le \omega$, and $\sigma_h$ is the corresponding hard bremsstrahlung cross section for photons of energy $E_\gamma > \omega$. Typical[1] results are presented in Table 1, where we have used parameters of (2), (5) and (6). As the infrared (IR) singular virtual one-loop corrections have not been included, $\sigma_s$ depends on a small fictitious photon mass $m_\gamma$ which has been chosen to be $m_\gamma = 10^{-6}$ GeV. For the reaction containing an electron in the final state, which receives a contribution from the $t$-channel photon exchange, we have introduced an additional cut $t_0 = -m_e^2(E_e^i - E_e^f)^2/(E_e^i E_e^f)$ in order to eliminate a possible divergence of the corresponding photon propagator. The results in Table 1 are in a sense a measure of the numerical stability of our calculation.

Our results for the fermion mass effects in $e^+e^- \to 4f$ and $e^+e^- \to 4f\gamma$ in the presence of the cuts specified in (8) are shown for several four–fermion final states in Table 2, where we have used the parameters of (4–7). In columns 3 and 4, we list respectively the results of [16] and [10] which were obtained in the massless fermion limit. The results of our calculation with nonzero fermion masses are shown in column 5. Wherever there is a substantial difference between the results of [16] and [10] and the present work, we present, in column 5, an additional entry representing the result of ours obtained with the initial fermion masses equal to zero and the final quark and/or charged lepton masses equal to $m_e$. We do not use an exact zero mass limit for the final state fermions as it is not easy to implement it in our kinematics routine, especially for the bremsstrahlung reactions. However, with the cuts of (8) the tiny value of $m_e$ should not play a numerically relevant role for the final state particles. With this substitution for the fermion masses, our results agree nicely with those of [16] and [10], except for $e^+e^- \to \nu_\tau\bar{\nu}_\tau\mu^-\mu^+\gamma$, where the difference amounts to a few standard deviations.

The reason for the difference between the results in the zero and nonzero mass cases can be traced back to the lower integration limit in the invariant mass of the lepton–antilepton or quark–antiquark pair, in particular $s\bar{c}$, which is different in both cases. The lower limit in $s_{ff'} = (p_f + p_{f'})^2$ is $s^0_{ff'} = 2E_fE_{f'}(1-\cos\theta(f,f'))$ in the zero mass case, whereas, in the nonzero mass case, it reads $s^{\min}_{ff'} = (m_f + m_{f'})^2$. For the $\mu^+\mu^-$ pair, $s^0_{\mu+\mu-}$ calculated with the cuts of (8) is a factor 6 smaller than the physical limit $s^{\min}_{\mu+\mu-} = 4m_\mu^2$. The lower the cut, the greater the cross section. The effect is enhanced by the $\sim 1/s_{\mu+\mu-}$ behaviour of the squared matrix element of $e^+e^- \to \nu_\tau\bar{\nu}_\tau\mu^-\mu^+$ which is usually mapped away in order to improve the convergence of the phase space integral. This argument explains also the $3\sigma$ discrepancy between the zero and nonzero mass cases for the radiative reaction $e^+e^- \to \nu_\tau\bar{\nu}_\tau\mu^-\mu^+\gamma$. Increasing the cut on $\theta(l,l')$ in (8) should result in a better agreement between zero and nonzero mass cases[2]. For the non–radiative channels also a comparison with `KORALW` [17] has been performed. Results agree well within the Monte Carlo errors.

[1] i.e., representatives of the classes I, II and III of processes considered below (see Table 2)

[2] We thank S. Dittmaier and other authors of [10] for a valuable comment on this point

**Table 2.** Mass dependence of cross sections at $\sqrt{s} = 190$ GeV (in fb) for three different classes of final states with the cuts of (8). The second entries in column 5 correspond to the initial fermion masses equal to zero and the final quark and /or charged lepton masses equal to $m_e$

| $\sigma$ (fb) | Final state of $e^+e^- \to$ | [16] ($m_f = 0$) | [10] ($m_f = 0$) | Present work ($m_f \neq 0$) | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | |
| I | $u\bar{d}e^-\bar{\nu}_e$ | 691.5(8) | 693.6(3) | 693.4(6) | |
| | $c\bar{s}e^-\bar{\nu}_e$ | – | – | 693.1(6) | |
| | $u\bar{d}e^-\bar{\nu}_e\gamma$ | – | 220.8(4) | 220.3(7) | |
| | $c\bar{s}e^-\bar{\nu}_e\gamma$ | – | – | 218.2(7) | |
| | $\nu_\mu\mu^+e^-\bar{\nu}_e$ | 227.0(3) | 227.5(1) | 227.5(2) | |
| | $\nu_\tau\tau^+e^-\bar{\nu}_e$ | – | – | 227.3(2) | |
| | $\nu_\mu\mu^+e^-\bar{\nu}_e\gamma$ | – | 79.1(1) | 79.0(3) | |
| | $\nu_\tau\tau^+e^-\bar{\nu}_e\gamma$ | – | – | 77.5(2) | |
| II | $u\bar{d}\mu^-\bar{\nu}_\mu$ | 667.4(8) | 666.7(3) | 666.7(4) | |
| | $u\bar{d}\tau^-\bar{\nu}_\tau$ | – | – | 666.0(3) | |
| | $u\bar{d}\mu^-\bar{\nu}_\mu\gamma$ | – | 214.5(4) | 213.8(3) | |
| | $u\bar{d}\tau^-\bar{\nu}_\tau\gamma$ | – | – | 209.3(5) | |
| | $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$ | 218.7(3) | 218.6(1) | 218.3(1) | 218.5(1) |
| | $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu\gamma$ | – | 76.7(1) | 75.1(2) | 76.6(2) |
| | $u\bar{d}s\bar{c}$ | – | 2015.3(8) | 2016(1) | |
| | $u\bar{d}s\bar{c}\gamma$ | – | 598(1) | 593(2) | 598(1) |
| III | $\tau^-\tau^+\mu^-\mu^+$ | – | 11.02(1) | 9.26(1) | 11.03(2) |
| | $\tau^-\tau^+\mu^-\mu^+\gamma$ | – | 6.78(3) | 5.32(3) | 6.62(5) |
| | $\nu_\tau\bar{\nu}_\tau\mu^-\mu^+$ | 10.121(40) | 10.103(8) | 10.05(1) | 10.095(10) |
| | $\nu_\mu\bar{\nu}_\mu\tau^-\tau^+$ | – | – | 8.529(6) | |
| | $\nu_\tau\bar{\nu}_\tau\mu^-\mu^+\gamma$ | – | 4.259(9) | 4.18(2) | |
| | $\nu_\mu\bar{\nu}_\mu\tau^-\tau^+\gamma$ | – | – | 3.167(7) | |
| | $\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu$ | 8.224(6) | 8.218(2) | 8.222(5) | |
| | $\nu_\tau\bar{\nu}_\tau\nu_\mu\bar{\nu}_\mu\gamma$ | – | 1.511(1) | 1.510(3) | |

The final states presented in Table 2 can be classified into three different classes I – III. According to classification of [18] class I corresponds to the CC20 family, class II to the CC11 family and class III includes the leptonic processes of the NC32 family. We see almost no mass effect for the tree level four–fermion reactions of class I and II. However, there is a difference of $\sim 1\%$ between cross sections of the radiative reactions $e^+e^- \to u\bar{d}e^-\bar{\nu}_e\gamma$ and $e^+e^- \to c\bar{s}e^-\bar{\nu}_e\gamma$ as well as between the results for $e^+e^- \to u\bar{d}s\bar{c}\gamma$ obtained in the mass limit and with the nonzero fermion masses. The effect is even larger ($\sim 2\%$) for $e^+e^- \to \nu_\tau\tau^+e^-\bar{\nu}_e\gamma$ and $e^+e^- \to \nu_\tau\bar{\nu}_\tau\mu^-\mu^+\gamma$.

The fermion mass effects become more pronounced for the reactions belonging to class III. We observe it here already for the tree level reactions $e^+e^- \to \tau^-\tau^+\mu^-\mu^+$ and $e^+e^- \to \nu_\mu\bar{\nu}_\mu\tau^-\tau^+$ where it amounts to about 15% and it becomes even stronger for the corresponding bremsstrahlung reactions. The reason for that is the presence of the virtual photon exchange Feynman graphs which introduce the $\sim 1/s$ behavior of the matrix element and the lack of the $W^\pm$-boson exchange graphs which are relatively insensitive to the fermion masses.

## 4 Summary

We have studied the fermion mass effects in the presence of cuts for several channels of $e^+e^- \to 4f$ and $e^+e^- \to 4f\gamma$ using the method of calculation elaborated in [12] and [11]. It has been shown that the fermion mass effects are typically of the order of 1% for the bremsstrahlung reactions in the semi-leptonic channels and they become even larger, of order 10% for some purely leptonic channels, the cross sections of the latter being much smaller however. For the non–radiative channels the deviations found are to a large extent not genuine mass effects but merely tell us that some of the so called "standard cuts" which have been utilized in the past do not respect the kinematical boundaries. This of course should be avoided in future.

Although the mass dependence of the individual channels seem not to affect the final LEP2 data analysis, where one expects an accuracy of about 1% in inclusive channels, they will certainly become relevant at a future $e^+e^-$ linear collider with the expected precision of 0.1%. Therefore, it seems to be much better not to neglect the fermion masses in calculations intending to match the high precision of the future experiments.

The cut independent results presented in Table 1 may serve as tests of MC generators which work with massless fermions especially in the collinear regions of phase space.

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